

## CALCULUS : Limit dan Turunan

1. Hitunglah limit dari fungsi berikut!

a.  $\lim_{x \rightarrow 1} \frac{x-2}{x^2+x+1}$

b.  $\lim_{x \rightarrow 2} \frac{x^2-4}{x-1}$

c.  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta^2}$

d.  $\lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{1 - \tan x}$

2. Carilah turunan pertama dari fungsi berikut!

a.  $f(x) = (2x^5 - x)(x^3 + 1)$

b.  $f(x) = (1 + x^2) x^3 e^x \ln x$

c.  $f(x) = e^{x^2}$

3. Dengan menggunakan aturan rantai, tentukan  $y'$  untuk  $y = (3x + 5)^2$ .

4. Carilah  $\frac{\partial f}{\partial x}$  dan  $\frac{\partial f}{\partial y}$  dari fungsi berikut!

a.  $f(x, y) = \frac{x-y}{x+y}$

b.  $f(x, y) = 2x \sin(x^2 y)$

## Jawaban

$$1. a. \lim_{x \rightarrow 1} \frac{x-2}{x^2+x+1}$$

$$= \frac{1-2}{1^2+1+1}$$

$$= -\frac{1}{3}$$

$$b. \lim_{x \rightarrow 2} \frac{x^2-4}{x-1}$$

$$= \frac{2^2-4}{2-1}$$

$$= \frac{0}{1}$$

$$= 0$$

$$c. \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta^2}$$

$$= \lim_{\theta \rightarrow 0} \frac{(\cos \theta - 1)(\cos \theta + 1)}{\theta^2 (\cos \theta + 1)}$$

$$= \lim_{\theta \rightarrow 0} \frac{\cos^2 \theta - 1}{\theta^2 (\cos \theta + 1)}$$

$$= \lim_{\theta \rightarrow 0} \frac{-\sin^2 \theta}{\theta^2 (\cos \theta + 1)}$$

$$= \lim_{\theta \rightarrow 0} \frac{-1}{\cos \theta + 1} \cdot \lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta^2}$$

$$= \frac{-1}{1+1} \cdot (1)^2$$

$$= -\frac{1}{2}$$

$$d. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \tan x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \frac{\sin x}{\cos x}}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos x - \sin x} \cdot \cos x$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} -\cos x$$

$$= -\frac{1}{\sqrt{2}}$$

$$2a. f(x) = (2x^5 - x)(x^3 + 1)$$

$$f'(x) = (10x^4 - 1)(x^3 + 1) + (2x^5 - x)(3x^2 + 1)$$

$$= (10x^7 + 10x^4 - x^3 - 1) + (6x^7 + 2x^5 - 3x^3 - x)$$

$$f'(x) = 16x^7 + 2x^5 + 10x^4 - 4x^3 - x - 1$$

$$b. f(x) = (1+x^2) \cdot x^3 \cdot e^x \ln x$$

$$= x^3 e^x \ln x + x^5 e^x \ln x$$

$$= 3x^2 e^x \ln x + x^3 (e^x \ln x + e^x \frac{1}{x})$$

$$+ 5x^4 e^x \ln x + x^5 (e^x \ln x + e^x \frac{1}{x})$$

$$= (e^x \ln x) (3x^2 + x^3 + 5x^4 + x^5) + x^2 e^x + x^4 e^x$$

$$c. f(x) = e^{x^2}$$

$$= \ln e \cdot e^{x^2} \cdot (2x)$$

$$= 2x e^{x^2}$$

$$3. y = (3x+5)^2$$

$$u = 3x+5$$

$$y = u^2$$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du} (u^2) \cdot \frac{d}{dx} (3x+5)$$

$$= 2u \cdot 3$$

$$= 2(3x+5) \cdot 3$$

$$y' = 18x + 30$$

$$4 \text{ a. } f(x, y) = \frac{x-y}{x+y}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left( \frac{x-y}{x+y} \right)$$

$$= \frac{(1)(x+y) - (x-y)(1)}{(x+y)^2}$$

$$= \frac{x+y - x+y}{(x+y)^2}$$

$$= \frac{2y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left( \frac{x-y}{x+y} \right)$$

$$= \frac{(-1)(x+y) - (x-y)(1)}{(x+y)^2}$$

$$= \frac{-(x+y) - (x-y)}{(x+y)^2}$$

$$= \frac{-2x}{(x+y)^2}$$

$$b. f(x, y) = 2x \sin(x^2 y)$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (2x \sin x^2 y)$$

$$= 2 \sin x^2 y + (2x) \cdot (2xy) (\cos x^2 y)$$

$$= 2 \sin x^2 y + 4x^2 y \cos x^2 y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (2x \sin x^2 y)$$

$$= 2x \cdot (x^2) (\cos x^2 y)$$

$$= 2x^3 \cos(x^2 y)$$